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THE COMPARATIVE ECONOMICS OF PASSIVE AND ACTIVE SYSTEMS*

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ABSTRACT

As the interest in solar energy applications for residential space heating grows, it becomes imperative to evaluate the economic performance of alternative designs. We concentrate on one passive design--the thermal mass storage wall. The economic performance of this design is examined and subsequently contrasted with one active design--the air collector/rock storage system. Architectural design criteria, solar performance characteristics, and the incremental solar cost of each design is briefly reviewed. Projections of conventional energy prices are discussed, along with the optimal sizing/feasibility criterion employed in the economic performance analysis. In addition, the effects of two incentive proposals--income tax credits and low interest loans--upon each design are examined. Results are reported on a state-by-state basis for the U.S., with major conclusions summarized for each design. It is generally the case that incentives greatly enhance the economics of both system designs, although the contrast is greater for the passive design. Also, against the less expensive conventional fuels (natural gas and heating oil) the passive design was shown to offer a more cost effective alternative than the active system for most states.

INTRODUCTION

As the interest in solar energy applications for residential space heating grows, it becomes imperative to evaluate the economic performance of alternative designs. This study describes another phase of our on-going efforts to examine solar feasibility in new home construction on a nationwide basis for the U.S. The first phase dealt exclusively with active solar systems, [1,2] while the current phase also includes passive solar concepts. These include (1) thermal storage wall, (2) roof pond, (3) direct solar gain, and (4) solar greenhouse (solarium).

In this paper we will concentrate on only one basic passive design -- the thermal storage wall. The economic performance of this design (with a night insulation option) will be examined and subsequently contrasted with one basic active design -- the air collector/rock storage system. Discussion of the results contained in this paper will exclude the other solar concepts being examined in the larger study. Since there exists several

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publications [1-4] on our past analysis of active solar systems, detailed description of the air collector/rock storage design and its economic performance will be omitted from this paper. Only that information necessary to understand the comparative economics of that system vis-a-vis the passive design will be highlighted.

The economic performance of these two basic designs (passive thermal storage wall plus an active air collector/rock storage system) is evaluated on a state-by-state basis. The section on methodology briefly reviews the architectural design crtieria, solar performance characteristics, and the incremental solar cost of each solar design, with emphasis placed on the thermal storage wall. Also included in this section is a discussion of conventional energy costs, as well as the optimal sizing/feasibility criterion employed in the economic performance analysis. In the third section, nationwide feasibility results are reviewed for each design. In addition to contrasting the solar designs themselves, the effects of two incentive proposals -- the proposed National Energy Act (NEA) income tax credits and low interest loans -- upon each design are examined. Finally, major conclusions are summarized in section four.

METHODOLOGY

There are essentially five basic steps employed in our evaluation of the economic performance of solar systems/designs. First, architectural design parameters for a standard home and solar system are established. Second, the physical performance of that system in various locales is estimated using a computer simulation code based upon a solar load ratio (SLR) correlation [5-7]. The solar performance characteristics (glazing area and storage volume) obtained from the simulation model are used to develop costs of providing alternative quantities of heat (solar fraction) for each locale. Fourth, the costs of providing heat through conventional means (natural gas, heating oil, and electricity--both resistance and heat pumps) are projected for each locale in the analysis. And finally, the potential for solar installations is evaluated through our economic performance analysis.

Solar Design

A standard home design* (approximately 1500 ft^2)† is being used throughout the analysis to allow inter-regional comparisons. Moreover, a 'tract' home concept and common building materials were assumed (Fig. 1 and Table I). This makes possible our examination for the potential of solar energy in

*The solar home design portrayed (Fig. 1) was developed by Burns & Peters, an architectural firm located in Albuquerque, New Mexico.

†We have chosen to use units common to the U.S. throughout this paper because of the geographical nature of analysis.

r idential space heating applications for a majority of new home buyers in th ted States.

I tion to incorporating traditional consumer preferences in the solar designs, minimum owner operating requirements were made part of the r system designs. Table II contains a brief description of the add-on solar components for the passive design. Detailed architectural characteristics of the thermal storage wall (in this case masonry or Trombe) concept are displayed in Fig. 2. For the air collector/rock storage system, collectors are mounted on a south facing pitch roof, with an insulated duct and rock storage bin strategically placed within the house to minimize add-on solar costs [8].

Solar Performance

The modified solar-load ratio (SLR) correlation procedures developed by Los Alamos Scientific Laboratory [6,7] were utilized to estimate solar performance given the parameters of the above solar system designs. This procedure is capable of treating several design parameters as variables: i.e., nominal building heat loads, glazing type, number of glazings, glazing area, storage volume, and storage type. Regional variability in weather patterns are taken into account in the performance computations. The modified SLR performance correlations are used to determine the glazing (collector) area required to achieve given solar fractions for the specific solar design under analysis. The ratio of glazing (collector) area to storage volume was held constant for each of the solar systems to ease the computational burden and limit the almost infinite construction design possibilities.*

For the Trombe wall design (both with and without night insulation) an 18 thick masonry storage volume with double glazing is used in the solar performance analysis [7]. Mean air temperature is kept at 70° F, with a 5° F temperature swing allowed -- auxiliary heat required when the interior temperature drops below 65° F, and excess heat purged when the interior reached 75° F. System performance measured by the glazing area required to provide a given solar fraction (ranging from 5 to 95 percent) was calculated for the Trombe wall design both with and without night insulation (R-9).

For the air collector/rock storage system a single pane flat plate collector with no selective absorption glazing was used in estimating its solar performance [8]. Insulated duct work with appropriate air handling equipment is used to transfer the air to either the home or rock storage.

*The constraint of a constant thermal storage to glazing area ratio has been relaxed in another phase of our efforts. The impacts of thickness variations in a Trombe wall design are examined in a paper describing that phase [9]. For all solar designs evaluated in this paper, a ratio was selected that appears to offer reasonable comfort.

Mean air temperature was set at 68° F, with a temperature swing of + or - 2° F allowed.

Building heat loads for each of these solar designs were very similar due to almost equivalent home designs. For the passive design, a standard 9 $Btu/DD/ft^2$ heat load factor was employed in the solar performance estimates,* while in the active system a 10 $Btu/DD/ft^2$ was assumed because of the addition of the south-facing wall.†

A comparison of the area requirements by solar fraction for each of the two basic solar system designs under analysis (Trombe wall, with and without night insulation, plus an active system) is contained in Fig. 3 (A and B). Figure 3A portrays that comparison for Albuquerque, New Mexico, while Fig. 3B displays the same comparison in Madison, Wisconsin. Table III summarizes glazing (collector) area required for representative solar fractions (portion of conventional heat replaced by solar) for the system designs in six representative sites.

Solar Cost

Every effort was made to construct realistic cost estimates for each solar design. In all cases we isolate the add-on solar components so that they may be priced independent of traditional home costs. In the active system, solar add-on components included collectors, roof supports, insulated ducts, an air handling system (fans, dampers, and controls), and a rock storage bin. For the passive designs the solar add-on components included the wall, glazing, and framing requirements. Credit was given for a portion of a normal wall replaced by the south-facing thermal mass storage unit.

The incremental solar cost of the two passive designs (with and without night insulation) used in our analysis are portrayed in Table IV.‡ These dollar figures represent solar costs across the nation.§ These dollar

*A nominal load without inclusion of the south-facing wall.

†This implies that the south wall section has a positive U-factor. Theoretical research and empirical validation by Bickle, Van de Meer, and Dexter [10,11] indicates that under certain conditions south walls may in fact have negative U-values over the heating season when the effects of solar radiation, building materials. exterior coloration, etc., are analyzed in a dynamic context. In this study we adhere to standard ASHRAE static heat load analysis, and thereby obtain a positive U-factor for the south wall section.

‡For the most part, these dollar estimates have been developed by Burns & Peters, an architectural firm located in Albuquerque, New Mexico.

§Costs were developed for a given locale then adjusted to reflect national dollar averages.

figures represent solar costs across the nation. In many solar system concepts, there are two cost components: a fixed cost which is pretty much independent of system sizes; and a variable cost associated with collector or glazing area requirements. However, no substantial fixed cost component was identified in these passive designs. This implies that all costs associated with each passive system can be stated strictly in terms of $\$/ft^2$ of glazing area once a storage to glazing area ratio is fixed. The credit for the wall is included in the cost figures displayed in Table IV. Since the costs displayed represent a national average, we subsequently adjust these materials and labor costs for each locale to account for regional variability in construction price indices and practices [12].

The incremental cost of the air collector/rock storage system has been developed previously [4]. To quickly review, we separate the system into a fixed collector-independent component and a variable collector-dependent component. The major portion of the rock storage bin, as well as the incremental insulated duct work and air handling system are treated as a fixed cost component -- i.e., their size and therefore installed dollar amount are assumed not to vary with changes in collector size. The variable cost component included the collectors, support requirements, and the collector area dependent portion of rock storage. These incremental solar costs were developed after a careful survey of manufacturers, a preliminary look at the HUD, ERDA, and EPRI Solar Demonstration Programs, and detailed discussions with solar engineering consultants and installers.

The national averages* employed in our analysis for the air collector/rock storage system were as follows: fixed cost component = \$2250, variable cost component = $\$13.50/ft^2$ of installed collector (glazing area). We have found it much more difficult to construct regional cost adjustment indices here to properly account for the transportation and distribution network that may be associated with a national product (collectors, insulated duct work, and air handling equipment) then was the case for the passive design where construction practices and costs are established on a regional basis. We therefore use the national averages for all locales in our economic performance analysis of the active system.

Representative solar costs, both the total (\$) and average† ($\$/10^6$ Btu heat provided), for each design are displayed for the six selected sites in

*It should be remembered that we are pricing a single-glazed flat plate collector with no selective absorptive material. As pointed out in a previous study our derived total installed costs per given solar fraction of heat provided is very similar to estimates made for other types of active systems (different collector design and glazing type/number). See Appendix A in [4].

†Average costs are stated in annualized terms. The computation formula is given as a footnote to Table V and in a following section on optimal sizing. A more complete explanation can be found in [13].

Table V. As evident, the total installed solar costs for the passive design (with and without night insulation option) increase at an increasing rate (all costs are variable). Also noted is the inability for the Trombe storage wall (<u>without</u> night insulation) to supply more than a given fraction (.60) of total annual heat load requirements. The impact of night insulation can be clearly seen. Even though on a $\$/ft^2$ basis night insulation is measurably more expensive (Table IV) than without, the solar performance of such a design in terms of glazing area required to meet a given solar fraction more than compensates for the add-on cost, thus reducing total delivered costs ($\$/10^6$ Btu) for an equivalent solar fraction. The impact of night insulation is greater as the climatic conditions become more severe (contrast between Albuquerque, New Mexico and Madison, Wisconsin). Finally, it should be pointed out that the total and average cost of the active system in relation to the passive designs (with and without night insulation) varies according to solar fraction and climatic conditions. For solar fractions of .60 or less, total installed costs of the Trombe wall concept fitted with night insulation is less than the air collector/rock storage system.

<u>Conventional Energy Costs</u>

Although we are examining many alternative energy futures, we utilize the proposed NEA as modified by the recent natural gas compromise in Congress to construct-projected fuel costs.* A 1977 state-by-state energy data base for natural gas (\$/MCF), heating oil (¢/gal), and electricity (¢/Kwh) prices has been constructed previously [2]. We then develop future price projections at the wellhead for natural gas and oil, at the meter for electricity, and add in a transportation, distribution, and marketing cost adjustment component (natural gas and heating oil only) to arrive at delivered or metered cost.

For natural gas price projections we use wellhead implications as reflected by the most recent natural gas compromise of the proposed NEA. This results in a fourth tier being established, with commenserately higher weighted average wellhead prices through 1985 and beyond. To arrive at metered cost, cost adjustment factors by locale reflecting transportation, storage, distribution, and marketing expenses are added to the national wellhead price in each year.

Heating oil price projections are based upon the original National Energy Plan (NEP) of April 1977† where a third world price tier (adjusted for inflation) for "new domestic production" is established. When the production

*For a more complete explanation of our projection procedures, see [2].

†Although there has been some changes as reflected through legislative debate for this portion of the initial NEP, no significant differences from the original are discernable in the proposed NEA. (The NEA evolved from the NEP, and many use the shortened names interchangeability to refer to the present legislative proposal package.)

from this new tier is combined with domestic production under the present two tier system and imported oil at $13.50 (1976 level), the "weighted" price of oil at the wellhead increases and is expected to asymptotically approach the world import price. Because of the entitlement program, the adjustment cost at each locale to reflect transportation, storage, distribution, and marketing is very similiar in most states. This results in a projected delivered cost at every locale being almost equivalent.

Electricity price projections stemming from the April 1977 NEP are more difficult to construct. Analysis performed by the Executive staff and White House consultants indicated a mixed estimate of future projections: increases, decrea[illegible] no change in real terms depending upon which region of the cou[illegible] was addressing. We have chosen to use a simple escalation factor [illegible]esent possible increases in the real price of electricity: 1 per[illegible] year. With 6% inflation, this implies a 7% annual rate of increase.

To construct equivalent delivered heating costs we transform the above fuel prices into a $\$/10^6$ Btu measure for each year. These figures are subsequently adjusted for furnace or heating equipment conversion efficiency. The efficiencies used in our present analysis reflect commonly used estimates: .75 for natural gas, .60 for heating oil, and 1.0 for electric resistance.* A heat pump offers an alternative to resistance heating. We account for delivered heating cost of that system by adjusting for the coefficient of performance (COP) at each locale. The seasonal COP varies between 1.75 and 2.75, depending upon climatic conditions.

Table VI displays the cost of delivered fuel for six representative sites used in the economic performance analysis. Both current and annualized prices are contrasted for 1978 and 1990. Note that nominal dollars are used. The computational procedures used in constructing both current and annualized price projections are given in footnotes to the table.

Optimal Sizing and Feasibility Without Incentives

We employ an equivalent set of criteria in our economic analysis of all solar energy system/designs. Reduced to its simplest form we evaluate a series of home heating systems that include a solar component, providing anywhere from zero to 100 percent of the required heat, to determine the economically optimal mix of solar and conventional back-up systems. The net present value (NPV) of a solar addition in concert with the fuel cost from a conventional furnace over the heating system life is maximized.

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*In empirical studies, J. McGraw of Applied Science and Engineering has found the average gas furnace efficiency in sample homes throughout the western states is only about 30-35%. Apparently, new furnace installations of the conventional type are not much better, which implies the effective cost of natural gas is more than twice what we use for the research reported here. This would significantly boost the economic desirability of solar systems as compared to this widely-used home heating fuel.

This is exactly equivalent to minimizing the cost of delivered heat to the home over a specified life time. The following discussion develops more formally the life cycle cost criteria that serves as the basis for our economic performance analysis.

We define the relevant variables as follows:

r = the real rate of interest

AIR = the annual rate of inflation

i = the nominal discount rate = r + AIR

VC = variable costs associated with each square foot of collector (collector plus storage)

FC = fixed costs (collector independent)

P_t = cost of back-up heat per 10^6 Btu (adjusted for furnace efficiency)

A = collector area in square feet*

F = fraction of space heating requirements to be provided by solar energy

LOAD = 10^6 Btu required per year

t = year

T = system life (30 years assumed for this paper)

CR = capital recovery factor

$$= \frac{1}{\sum_{t=0}^{T} \left(\frac{1}{1+i}\right)^t} = \frac{i}{1 - \left(\frac{1}{1+i}\right)^t}$$

OP = operation and maintenance expenditures expressed as a percent of total equipment investment

From the LASL solar performance programs [6,7], we know the relationship between collector area and the fraction of solar heat provided, A(F). One would like to size a solar system so that the present discounted value of total life cycle costs (including initial costs, back-up fuel costs, and

*The storage to glazing area ratio is fixed here for both the active and passive thermal storage system concepts.

operation and maintenance charges) are minimized.* Therefore, one should minimize

$$VC \cdot A(F)+FC + \sum_{t=0}^{T} \left(\frac{1}{1+i}\right)^{t} P_t \cdot LOAD \cdot (1-F)$$
$$+ \sum_{t=0}^{T} \left(\frac{1}{1+i}\right)^{t} OP \cdot \left[VC\ A(F)+FC\right] \qquad (1)$$

with respect to the fraction (F) of solar heat provided.† This cost minimization implies that

$$VC \cdot (dA/dF) - \sum_{t=0}^{T} \left(\frac{1}{1+i}\right)^{t} \cdot P_t \cdot LOAD + VC \cdot (dA/dF)$$
$$x\ OP \cdot \sum_{t=0}^{T} \left(\frac{1}{1+i}\right)^{t} = 0\ , \qquad (2)$$

which is the derivative of Eq. (1) with respect to F set equal to zero. Factoring and rearranging terms, Eq. (2) can be restated as

$$VC \cdot (dA/dF) \cdot \left[1+OP \sum_{t=0}^{T} \left(\frac{1}{1+i}\right)^{t}\right]$$
$$= \sum_{t=0}^{T} \left(\frac{1}{1+i}\right)^{t} P_t \cdot LOAD\ . \qquad (3)$$

*An equivalent expression is the maximization of new present value.

†We ignore the installation cost of the back-up heating system because such a system is required with or without solar heating and so cancels out in making cost comparisons.

Dividing both sides of Eq. (3) by the term

$$\sum_{t=0}^{T} \left(\frac{1}{1+i}\right)^{t}$$

and noting that

$$1/ \sum_{t=0}^{T} \left(\frac{1}{1+i}\right)^{t} = CR,$$

Eq. (3), with additional manipulation, reduces to

$$\frac{VC \cdot dA}{LOAD \cdot dF} \cdot [CR + OP] = CR \cdot \sum_{t=0}^{T} \left(\frac{1}{1+i}\right)^{t} P_t \ . \tag{4}$$

If the fixed charge rate (FCR) is defined as FCR = CR + OP and the annualized price ($\overline{P}$) of the conventional energy source is defined as

$$\overline{P} = CR \sum_{t=0}^{T} \left(\frac{1}{1+i}\right)^{t} P_t \ ,$$

the condition for optimal sizing becomes

$$\left[\frac{VC \cdot dA}{LOAD \cdot dF}\right] \cdot FCR = \overline{P} \ . \tag{5}$$

Equation (5) implies that the solar system will be optimally sized when the marginal cost of obtaining the incremental unit increase in the annual solar fraction (by increasing the collector area) is just equal to the annualized equivalent of the conventional energy price. The A's are known for values of F between .05 and 1.0 in .05 increments from the LASL solar performance simulations. We can calculate the change in A (ΔA) for the corresponding change in F where $\Delta F = .05$. Thus, the optimum value of F and consequently the optimal collector area is determined where:

$$\left[\frac{VC \cdot \Delta A}{LOAD \cdot 05}\right] \cdot FCR = \overline{P} \ . \tag{6}$$

Feasibility, however, is not insured by this process. Rather, given an annualized price of energy, collector area will be optimally sized. To check for feasibility one must compute the optimum percentage of space heating requirements to be met by solar energy (fraction of solar heat provided, F*) and the associated collector area (A*) and using that percentage, calculate the average annualized cost of delivered heat ($\overline{P}_h$).

The average annualized cost of delivered heat is determined by simply summing the total annualized cost of the optimally sized solar system with the annualized cost of auxiliary energy and dividing this sum by the total Btu heating load of the home.

$$\overline{P}_h = \frac{[VC \cdot A^* + FC] \cdot FCR + \overline{P} \cdot LOAD \cdot (1-F^*)}{LOAD} \tag{7}$$

or

$$\overline{P}_h = \frac{(VC \cdot A^* + FC) \cdot FCR}{LOAD \cdot F^*} \cdot F^* + \overline{P}(1-F^*) \text{ ,} \tag{8}$$

which can be interpreted as the weighted sum of the average annualized cost of the solar system alone (P_s, the bracketed term in equation 6) and the annualized cost of the conventional back-up fuel (P). Thus,

$$\overline{P}_h = \overline{P}_s \cdot F^* + \overline{P} \cdot (1-F^*) \text{ ,} \tag{9}$$

where F* and (1-F*) serve as the weights on the solar and conventional costs, respectively. If this annualized cost of delivered energy is less than or equal to the annualized cost of back-up heat ($\overline{P}_h \leq \overline{P}$), then the percentage of space heating requirements to be met by solar energy determined above is correct, and therefore solar energy for residential space heating is feasible. If, however, the annualized cost of back-up heat is less than the annualized cost of delivered energy with solar, then the solar energy system is not feasible and we set the solar fraction equal to zero. Note that if we are interested in current cost comparisons, the current price of alternative energy can be substituted for $\overline{P}$.

As F increases from .05 to 1.0 for each site, A increases at an increasing rate, making ΔA a monotonically increasing function. This means that total variable cost (VC·A) is also increasing monotonically, whereas FC by definition is constant. We obtain traditional cost curves as depicted in Fig. 4,† where MC_s and AC_s represent the annualized cost in 10^6 Btu of a

†This figure refers to the air collector/rock storage system. Subsequent discussion and Fig. 5 describe the situation for passive designs.

specific solar system. It is important to note, however, that the annualized cost of delivered energy ($\overline{P}_h$) is what determines feasibility, not P_s which is the average annualized cost of so . energy without regard to back-up fuel costs. Remember that $\overline{P}_h$ is given by the weighted average sum formula (9), or again

$$\overline{P}_h = \overline{P}_s \cdot F^* + \overline{P}\ (1-F^*)\ ,$$

where F* is the optimally determined solar fraction. Thus, as $\overline{P}$, the annualized price of back-up energy, increases from $\$5.00/10^6$ Btu to $\$9.00/10^6$ Btu, the shape of the $\overline{P}_h$ curve changes as shown in Fig. 4, whereas $\overline{P}_s$ remains fixed regardless of the value of $\overline{P}$. When $\overline{P}$ just equals the minimum value on $\overline{P}_s$, the minimum of $\overline{P}_h$ exactly coincides. In the figure this occurs when $\overline{P} = \$7.50/10^6$ Btu. For any value of $\overline{P}$ below \$7.50, the average annualized cost of delivered heat with solar will be greater than the annualized price of back-up energy ($\overline{P}_h > \overline{P}$), so it would be uneconomical to invest in a solar energy system. However, as $\overline{P}$ rises above \$7.50, not only is feasibility obtained ($\overline{P}_h < \overline{P}$) but the optimal system size increases. Thus, the system should be sized to provide approximately 43% solar when $\overline{P}$ = \$7.50 and 52% when $\overline{P}$ = \$9.00. In passive systems, we have found no fixed cost components, so total initial costs are equivalent to total variable costs [TC = VC* A(F)]. The optimality condition is still given by equations (5) and (6), but the delivered cost of heat is now defined as

$$\overline{P}_h = \frac{(VC \cdot A^*) \cdot FCR}{LOAD \cdot F^*} \cdot F^* + \overline{P}(1-F^*) \tag{10}$$

or as

$$\overline{P}_h = \overline{P}_s \quad F^* + \overline{P} \cdot (1-F^*)\ . \tag{9}$$

Again, $\overline{P}_s$ is the average cost curve for solar but it is no longer U-shaped. Representative average cost, marginal cost, and delivered cost of heat curves for the passive solar designs are shown in Fig. 5. The minimum of any delivered heat cost curve corresponds to the intersection of the marginal cost curve and the appropriate value of $\overline{P}$. In addition, the delivered cost will equal the average cost curve where $\overline{P}$ and AC intersect. The forme condition obtains with or without fixed costs, whereas the latter condition only occurs when FC = 0 or, in the event of FC > 0, only when P > min [AC].

Optimal Sizing with Incentives*

The above process would ensure an optimally sized system if no incentives existed. However, once the incentives (proposed NEA income tax credits assumed in following discussion) are taken into account, the economics of the solar system changes because they effectively involve a reduction in both the average and marginal costs of the system. This implies that for an unchanged value of $\overline{P}$, it would be worthwhile to increase the collector area and solar fraction beyond the optimal size determined without incentives. A simple example should illustrate this point. Suppose that a refundable income tax credit can be applied to 20% of the total initial solar system cost without an upper limit.† The problem then becomes, minimize total life cycle costs [after equation (1)]

$$[VC \cdot A(F) + FC]\ [1-.20] + \sum_{t=0}^{T} \left(\frac{1}{1+r}\right)^{t} P_t \cdot LOAD(1-F)$$

$$+ \sum_{t=0}^{T} \left(\frac{1}{1+r}\right)^{t} OP$$

$$x\ [VC \cdot A(F) \cdot FC] \cdot [1-.20] \qquad (1A)$$

with respect to the solar fraction (F). This yields an optimality condition given by,

$$.8 \left[\frac{VC \cdot dA}{LOAD \cdot dF}\right] \cdot FCR = \overline{P}\ .$$

*The discussion presented here assumes an income tax credit incentive option. We will exclude discussion on a low interest loan form of incentive, but only note that the same general conclusions apply: i.e. the MC curve will shift downwards by a constant percentage amount for all solar fractions.

†This simplifies the actual structure of the incentives which in the latest version of the proposed NEA, for example, are 30% on the first $1500 and 20% on the next $8500 for a maximum credit of $2150 on a $10,000 system; any system cost above $10,000 does not benefit from additional incentives. The example above is structured for illustrative purposes.

Upon inspection of equations (1a) and (1), one can see that the marginal cost of the solar system with the incentive is 80% of the marginal cost without such an incentive. The only way to satisfy the condition given by (1a) is to increase dA/dF above its optimal value as given by (5); and such an increase can only be obtained by sizing the system to meet a higher solar fraction.* This is shown in Fig. 6† where the marginal cost curves are depicted with and without the incentive.

Figure 6 shows that the marginal cost with incentives (MC_W) is less than the marginal cost without incentives ($MC_{w/o}$) for any given solar fraction F. Since the annualized price of back-up energy ($\overline{P}$) is not affected by the incentives, $\overline{P}$ is depicted as a horizontal line. Sizing the system at the old fraction $F_{w/o}$ under the incentive plan would imply that MC_w (as given by point X) is less than $\overline{P}$ which is less than optimal. One would therefore size the system to provide a solar fraction F_w at which point $MC_w = \overline{P}$, and life cycle costs are at a minimum. The average cost of providing this newly optimized solar fraction is also lower than would be the case without incentives. This implies that economic feasibility is obtained at a lower value of $\overline{P}$, which with rising energy prices corresponds to an earlier point in time.

RESULTS

In this section, we present only selected results from our economic performance analysis. Excluded for all solar designs are comparisons with heating oil and electric heat pumps.

As stated in the Introduction, we are interested in the individual economic performance of each solar design in addition to its comparative economic performance among systems. As part of this analysis the effects of two

*dA/dF can be defined as the inverse of the marginal product of collector area (MP), i.e., $dA/dF = 1/MP_A$ where MP_A indicates the increase in solar fraction (dF) due to an incremental increase in collector area (dA). Since $d(MP_A)/df < 0$, the marginal product of collector area becomes smaller as the solar fraction is increased. This implies dA/dF will only increase if F increase.

†This figure is applicable to both active and passive system designs.

alternative incentive options are evaluated: the NEA income tax credits,* here assumed applied equally to passive as well as active systems, and low interest loans.† The impact of night insulation in the economic performance of the Trombe wall concept is carefully noted. The Trombe wall concept is contrasted with the one active system design under review here, the air collector/rock storage design.

In addition to including maps portraying several aspects of the economic feasibility results (Maps 1-12), we include a table (Table VII) highlighting selected financial indicators for the comparative evaluation of solar systems: these include mortgage payback period, years to positive savings, and net present value.

Basic solar energy cost and conventional energy pricing assumptions employed in the economic performance analysis have been discussed previously. Computational formulaes and selected values for various locales can be found in Tables V and VI and in the discussion on optimal sizing.

The nine cases reported below can be summarized as follows:

Case 1 - Air Collector/Rock Storage; No Incentives
Case 2 - Air Collector/Rock Storage; NEA Income Tax Credits
Case 3 - Air Collector/Rock Storage; Low Interest Loans
Case 4 - Air Collector/Rock Storage; NEA Income Tax and Credits Low Interest Loans
Case 5 - Trombe Wall without Night Insulation; No Incentives
Case 6 - Trombe Wall with Night Insulation; No Incentives
Case 7 - Trombe Wall with Night Insulation; NEA Income Tax Credits
Case 8 - Trombe Wall with Night Insulation; Low Interest Loans
Case 9 - Trombe Wall with Night Insulation; NEA Income Tax Credits and Low Interest Loans

In all nine cases comparisons to both natural gas and electric resistance alternative fuel types are made.

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*The latest version from the proposed NEA of the solar income tax credits is the specific form under review: 30 percent of the first $1500, 20 percent on the next $8500, with a maximum of $2150 for systems $10,000 and over. The tax credits are assumed to begin in 1978 and continue at the same levels through 1984, at which point they are terminated for 1985 and following years.

†The specific value employed in our low interest loan incentive is 3 percentage points: that is the government would subsidize the difference between the going mortgage rate and the rate paid by consumers under this program at a rate 3 percentage points below the mortgage rate. In the specific analysis reported here a mortgage rate of 9.5 percent is employed with consumer loans available for the solar components at 6.5 percent.

For the air collector/rock storage active system (Case 2)* in only one state--Maine--does it prove economic to install solar when natural gas is the alternative fuel. The price of natural gas remains below the cost of solar through 1990, despite its (natural gas) rather rapid rate of increase under the recent compromise. Without the proposed NEA tax credits (Case 1) there would be no states included in the feasible set.

For the low interest loan option alone (Case 3) against the natural gas fuel alternative, in only three states does it prove economic to install solar: Maine, New Hampshire, and Vermont. When this option is combined with the proposed NEA income tax credits (Case 4), a number of additional states join the feasible set. These states are located in New England and the northern Rocky Mountain, Great Plains, and Midwest regions of the US (Map 1).

If the Trombe wall without night insulation design (Case 5) is contrasted with natural gas, only in two states does it appear economic to install such a design in a new home: Maine in 1978 and Idaho in 1983. Map 2 portrays this relatively sparse feasibility pattern. Note there are no incentives included in the economic performance evaluation of this case.

By the addition of night insulation to the Trombe wall concept (Case 6), some additional states join the feasibility set when natural gas is the alternative fuel. This pattern is displayed in Map 3. Except for North Carolina, the additional states are located in New England. Here again, the incentives are not yet part of the economic performance analysis.

Inclusion of the proposed NEA income tax credits† in the solar cost component gives rise to a much greater number of states portraying economic feasibility against natural gas. As seen in Map 4, the general location of those states achieving solar competiveness is the New England, Midwest, Plains, and Western regions of the US. A few of the feasible states are also located in the Southeast region. By contrasting Map 4 with Map 3, it can be seen that the year of feasibility is moved forward for those states appearing in both. Generally speaking the first year of feasibility is 1978, except for those states in the Plains and Southwest regions where feasibility is delayed to the period between 1981 and 1984. In the states where solar was not shown to be competitive, the cost of this passive

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*For past discussions on the economic performance of active systems see [1, 2, 5, and 9-11].

†We assume here that the add-on solar costs associated with passive designs are treated the same as those proposed for active systems. That is, full credit is given our computations of the additional cost incurred for the passive designs. An alternative is to allow only a partial credit in the sense that not all of the add-on solar cost can be used in tax credit computations. The impact of such a tax credit system has been evaluated, but the results are not included in this paper.

design (Trombe Wall with night insulation) even with the proposed NEA tax credits remains higher than projected natural gas prices.

If low interest loans are substituted for the NEA tax credits, a larger number of states enter the feasibility set (Map 5) against the natural gas alternative. In addition, in a number of states feasibility is achieved at an earlier date than when the NEA income tax credit form of incentive was used. The additional states (over and above those displayed in Map 4) are generally located in the Ohio River valley and Northeast regions. This indicates that the particular form of low interest loans evaluated here performs better than the proposed NEA income tax credit option.

When both incentive options are combined, as has been indicated in recent Congressional debate, only in Florida and Louisiana is the Trombe Wall with night insulation design not economically competitive. Moreover, the year of feasibility is 1978 for all states (Map 6), except Florida and Louisiana. Thus, use of both incentives appears to be potentially beneficial to most of the US.

The above reviewed results clearly shows that the proposed NEA income tax credits have a greater impact for passive designs than for active systems. That is, for the air collector/rock storage system inclusion of tax credits gave rise to only one state portraying solar competitiveness against natural gas. Comparison of Maps 3 and 4 show that a significant number of additional states achieve solar economic parity against the natural gas alternative when given a tax credit. Similar patterns emerge when low interest loans are substituted for the tax credits, or are used in combination with them: solar feasibility is promoted noticeably less for the active design than for passive when natural gas is the alternative fuel (Maps 4, 5, and 6). However, potential energy savings is diminished somewhat due to the relative low (below 40 percent even with inclusion of both incentive options) solar fractions when compared to what solar fractions (35 to 60 percent) are present when active systems are examined.*

A somewhat different picture emerges when electric resistance is used as the alternative fuel type against which the solar designs must compete. For the air collector/rock storage active system, solar competitiveness is achieved in most US states (Map 7) when the proposed NEA income tax credits (Case 2) are applied against the solar costs. The Northwest and the lower Mississippi and Ohio River valleys are excluded. (It is generally the case that in those states either electricity prices are relatively low and/or

*When active systems have been forced into an economically competitive position against natural gas, the optimal solar fraction has usually ranged from 35-60 percent. This, of course, is due to the role of the fixed cost component in active systems. Thus, energy savings would be greater for the US if active systems were deployed over pure passive designs. However, the economics, as is being shown here, lean heavily to passive designs in most regions of the US.

low total heat loads are present.) Moreover, the year of feasibility is 1978 except for South Carolina (1982) and Texas, Utah, and Vermont (1983). As evidenced in Map 7, the solar fraction is at least 45 percent (except Idaho at 40 percent) for all states displaying solar feasibility. The higher fractions are generally in the Southwest where isolation is high, or along the Eastern Seaboard states where electricity prices are relatively high. Although not shown here, when NEA income tax credits are not included (Case 1) in the economic performance analysis, feasibility will occur at a later date with smaller fractions. In addition a few states, located principally in the South and Southwest where heat loads are lower, drop from the feasible set.

The Trombe wall without night insulation design (Case 5) is able to compete against the electric resistance alternative in all states except Washington without the inclusion of incentives. As portrayed in Map 8 the solar fractions range from 15 to 40 percent in all states except Arizona, California, and South Carolina--the solar fractions in these states being 5 to 10 percentage points higher. States in the Midwest and Plains region generally have lower solar fractions then the remainder of the country. In all states, excepting Washington, the year of demonstrated solar competitiveness is 1978. The results reported here do show that it is cost effective now (given our solar costs and alternative fuel prices projections) to employ some passive designs in new home construction throughout the country.

When night insulation is added to the Trombe wall design (Case 6), there is an incremental increase in optimal solar fraction for all but two states--these being Louisiana and Oregon (with the state of Washington still excluded from economic feasibility). Map 9 contains a summary of the incremental change for all states. As can be seen, in the majority of states the incremental change is 15 percentage points or greater. It is primarily in the Ohio River valley states (plus Arizona and California) that the incremental change is less at 5 to 10 percentage points. The highest change occurs in the Rocky Mountain, Northern Great Plains, and New England regions. Thus as discussed earlier, the more severe the climate, the more important becomes the use of night insulation in the achievement of maximum economic performance. The results are essentially indicating that for a similar dollar outlay, the consumer can purchase a more efficient solar system by adding night insulation to a Trombe wall concept.

[Because we have constrained the maximum allowable glazing area to account for permissable tract home characteristics (8' x 56' south-facing wall maximum), the inclusion of incentives in the economic performance analysis will not increase optimal solar fraction in those states where the constraint is binding. Therefore, in the remaining discussion of results for individual cases (Cases 7, 8, and 9) it will be seen that no visible change occurs in some northern states. However, in all cases the dollar cost paid by consumers in obtaining this maximum solar fraction will be appreciably lowered.]

With the inclusion of proposed NEA income tax credits in the economic performance analysis, further additions are made to the optimal solar fraction in a number of states (see above for discussion of why all states do not show increases). These incremental increases are portrayed in Map 10. As evident, tax incentives are important for they increase substantially the potential energy savings in new home construction (higher solar fractions) and lower the total system costs paid by the consumer.

If low interest loans were to be used instead of income tax credits, a greater number of states would achieve higher solar fractions (Map 11). In addition to more states, it is almost always true that the fractions are greater than under tax credits--as is evidenced by contrasting Map 11 with Map 10. As was pointed out when natural gas was the alternative fuel type, the specific low interest incentive employed in our analysis (3 percentage points less than the assumed 9.5 percent mortgage rate) performs better than the proposed NEA income tax credits.

As shown in Map 12, a combination of both incentives when compared to the electric resistance alternative performs better than either alone. The combination of incentives has the effect of pushing more states to their physical maximum. This will of course result in even greater energy savings than under either incentive option individually.

Against the electric resistance alternative, the difference between active and passive designs is less noticeable than in the comparisons with natural gas. It is still the case that incentives do have a significant impact on both system types, but here the difference between Case 2 and Case 7 is not nearly as dramatic as was true when contrasting the natural gas alternative (Map 4). It is true, however, that the passive design (Trombe wall with night insulation) is economically competitive in all states, whereas for the air collector/rock storage system seven states were excluded from the feasible set. As stated under the natural gas alternative comparison, the active system does provide a higher solar fraction than the Trombe wall concept.

Table VII contains some selected financial indicators for six of the nine cases at six representative locations. As stated earlier, we are highlighting results from the passive design.

SUMMARY

The following points serve to summarize the basic findings from our analysis. As cautioned in previous work [2], economic feasibility is a necessary, but not sufficient, condition for large-scale market penetration. It establishes basic criteria, when combined with other information, which allow estimates of penetration to be made. The major conclusions are:

- With natural gas as the alternative fuel, the passive concept evaluated here offers more promise than the active system. This is true with or without inclusion of incentives, although either incentive option enhances economic performance for both designs.

- The addition of night insulation to thermal mass storage walls makes a significant difference: not only in the solar performance, but more importantly in the economic performance of this passive concept. In addition, the effectiveness of night insulation becomes greater as the severity of the climate increases.

- The potential use of passive solar (thermal mass storage wall) in residential space heating applications is measurably enhanced by incentives against all fuel types. This enhancement is especially evident in the natural gas and heating oil comparisons.

- The passive design evaluated in this paper is economically competitive against the electric resistance alternative in all but a few states. Moreover, on a life cycle cost basis this design is feasible today.

- Employment of the low interest loan incentive option gives rise to higher solar fractions than under the proposed NEA income tax credit option. The particular low interest loan incentive evaluated here reduces solar costs for the homeowner more than the tax credit does.

- Although the optimal solar fractions are generally low, the passive design offers one the opportunity to incorporate solar into a new home at costs much less than its active counterpart. This is because there are no discernable fixed costs, thereby allowing a simple movement from zero to 100 percent solar when evaluating economic feasibility.

- When both active and passive designs are shown to be cost competitive against alternative fuels, higher solar fractions will be associated with the active systems. This is due principally to the substantial fixed cost component of active systems, which forces one to achieve a given solar fraction before economic feasibility can be shown.

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TABLE I

CONVENTIONAL TRACT HOME DESCRIPTION

1) Typical Living Space Design -- 2 Zone

   - Public area -- living room, dining room, kitchen, den-study

   - Private area -- bedrooms, bathrooms

2) Living Space* -- 1496 ft^2

3) South Linear Exposure -- 50 ft

4) Two Car Garage

5) Minimum Lot Size (Frontage) -- 60 to 70 ft

6) Construction

   - Walls -- 2 x 4 wood frame

   - Floor -- slab on grade

   - Roof -- flat or pitched

   - U Factor† -- 562.5 Btu/Hr-°F or 9 Btu/DD/ft^2_R

*This is for a standard dimension (approximately 30' x 50') which could be altered to 27' x 56' to accommodate additional southern exposure. With an 8' maximum glazing height on a single story residence, maximum southern exposure is limited to 448 ft^2 (56' x 8'). The addition of direct gain clerestory windows could increase the transmitted solar energy, but this option is not considered in this particular configuration.

†This is exclusive of south wall.

TABLE II

SOLAR DESIGN ADDITIONS: MASONRY THERMAL STORAGE WALL

1) Load Bearing Foundation

- Poured for entire south wall length
- Supports concrete or water storage wall

2) Thermal Storage Mass

- 18" concrete masonry construction
- On-site poured concrete or tilt-up slab are other options

3) Thermocirculation Vents with Barometric Damper - Manual Override

- .074 ft /ft of wall length for each vent
- Solid wall -- no vents
- Trombe wall -- no reverse thermocirculation

4) Glazing

- Double-glazed tempered patio glass
- No special characteristics

5) Night Insulation

- Solar/load calculations with and without night insulation
  - Without - R = 1.75 (double glazing)
  - With - R = 9.0 (4 layer Kalwall design)
- Installed between wall exterior and glazings
- Night insulation down between 5 pm and 7 am

6) Framing

- Aluminum framing for glazing units

7) Header Trim or Overhand

- Installed to prevent Spring, Summer, and Fall overheating

TABLE III

REQUIRED GLAZING AREAS (FT^2) FOR REPRESENTATIVE SOLAR FRACTIONS
(SIX SELECTED SITES)

Solar System Design		Solar Fraction .15*	.30	.45	.60	.75
Trombe Wall - No Night Insulation						
Albuquerque	NM	74	163	293	482	844
Madison	WI	178	466	1038	-	-
Boston	MA	144	346	711	1500	-
Seattle	WA	100	250	519	1125	-
Charleston	SC	49	109	193	314	519
Omaha	NB	144	346	675	1350	-
Trombe Wall - Night Insulation						
Albuquerque	NM	71	110	180	276	422
Madison	WI	105	241	422	675	1125
Boston	MA	89	201	338	540	900
Seattle	WA	64	152	276	466	794
Charleston	SC	35	77	126	190	293
Ohama	NB	90	199	338	519	844
Air Collector/Rock Storage						
Albuquerque	NM	71	117	202	315	483
Madison	WI	147	251	450	725	1129
Boston	MA	132	221	389	615	948
Seattle	WA	101	177	339	583	984
Charleston	SC	54	90	157	243	363
Omaha	NB	128	213	374	590	912

*For Air Collector/Rock Storage Active system the first representative fraction is .20 instead of .15

TABLE IV

DETAILED COST BREAKDOWN: THERMAL STORAGE WALLS*

Component	Cost ($) Material	Labor	Total
Masonry Concrete 18"	2.72	3.81	6.53
Paint - 2 sides	.11	.33	.44
Glazing - Glass Double 2 3/16"	2.72	.82	3.54
Footing 16" Foundation	.82	.34	1.16
Header Trim or Overhang	.68	.68	1.36
Framing 4' x 8' = 24 ft_L	.41	2.45	2.86
Conventional Wall Credit			(2.27)
Total System			13.60
Night Insulation Kalwall Insul Curtain 4 layer R = 10.1	3.53	.82	4.35

*Dollar costs are for national averages

TABLE V

TOTAL ($) AND AVERAGE* ($/10^6 Btu) COST FOR REPRESENTATIVE SOLAR FRACTIONS (SIX SELECTED SITES)

SOLAR FRACTION

Solar System Design	.15** TC	.15** AC	.30 TC	.30 AC	.45 TC	.45 AC	.60 TC	.60 AC	.75 TC	.75 AC
Trombe Wall-No Night Insulation										
Albuquerque, NM	977	11.84	2155	13.06	3889	15.71	6388	19.35	11180	27.09
Madison, WI	2333	15.63	6114	20.46	13640	30.46	--	---	---	---
Boston, MA	2102	19.65	5066	23.68	10398	32.41	21951	51.31	---	---
Seattle, WA	1507	17.54	3767	22.43	7823	31.05	16950	50.46	---	---
Charleston, SC	529	13.70	1173	15.19	2077	17.94	3381	21.91	5592	28.98
Omaha, NB	1932	15.39	4656	18.55	9080	24.12	18160	36.17	---	---
Trombe Wall-Night Insulation										
Albuquerque, NM	898	11.39	1920	12.17	3148	13.31	4819	15.28	7379	18.72
Madison, WI	1814	12.73	4180	14.66	7314	17.20	11703	20.52	19505	27.36
Boston, MA	1727	16.91	3892	19.05	6519	21.27	10431	25.53	17385	34.04
Seattle, WA	1279	15.94	3017	18.80	5479	22.77	9258	28.85	15793	39.38
Charleston, SC	800	13.56	1090	14.79	1794	16.22	2703	18.33	4172	22.64
Omaha, NB	1598	13.33	3525	14.70	5993	16.66	9220	19.23	14982	24.99
Air Collector/Rock Storage										
Albuquerque, NM	3716	26.59	4059	24.21	5276	20.97	6893	20.55	6297	22.18
Madison, WI	4489	22.20	5977	15.17	8824	19.40	12760	21.04	18541	24.46
Boston, MA	4274	29.50	5548	25.53	7952	24.40	11186	25.74	15951	29.36
Seattle, WA	3830	33.67	4918	28.82	7236	28.27	10728	31.44	16466	38.60
Charleston, SC	3158	60.41	3673	46.84	4632	39.38	5862	37.38	7580	38.67
Omaha, NB	4217	24.80	5433	21.31	7737	20.23	10828	21.23	15436	24.21

TC = Total   AC = Average Cost

*The average cost is defined as follows:

$$AC = FCR \times \left[\frac{VC \times A(F) + FC}{LOAD \times F}\right] \quad \text{where}$$

FCR = fixed charge rate = CR + OP

VC = variable cost ($/$ft^2$)

A(F) = glazing (collector) areas required to obtain F

F = solar fraction

FC = fixed cost ($)

LOAD = Btu requirements for the home

AC = average cost of solar heat provided for given F

CR = capital recovery factor = $\dfrac{i}{1-\left(\frac{1}{1+i}\right)^T}$

OP = operating and maintenance expense (expressed as a percent of solar cost)

i = r + AIR

r = real rate of interest

AIR = annual inflation rate

Values used in the derivation of these average cost figures are as follows:

r = .035

AIR = .06

i = .095

T = 30

CR = .102

OP = .005 for Trombe Wall design w/o night insulation

.01 for Trombe Wall design with night insulation

.015 for Air Collector/Rock Storage system

**For Air Collector/Rock Storage active system the first representative fraction is .20 instead of .15.

TABLE VI

COST OF DELIVERED FUEL* ($\$/10^6$ BTU) BY FUEL TYPE--CURRENT & ANNUALIZED** PRICES IN 1978 AND 1990 DOLLARS
(Six Selected Sites)

	Natural Gas				Heating Oil				Electric Resistance				Heat Pump			
	Current		Annualized		Current		Annualized		Current		Annualized		Current		Annualized	
Location	78	90	78	90	78	90	78	90	78	90	78	90	78	90	78	90
Albuquerque, NM	2.64	10.40	8.05	20.08	6.21	13.79	12.23	25.34	12.15	27.55	24.88	56.41	6.74	13.55	12.18	24.52
Madison, WI	3.73	12.58	10.01	24.03	6.05	13.47	11.94	24.75	12.20	27.66	24.98	56.63	8.81	17.72	15.93	32.06
Boston, MA	5.06	15.26	12.43	28.88	6.44	14.25	12.64	26.17	15.97	36.20	32.69	74.12	9.67	19.46	17.50	35.21
Seattle, WA	4.25	13.63	10.96	25.93	6.36	14.09	12.50	25.88	5.24	11.88	10.73	24.32	2.92	5.87	5.28	10.63
Charleston, SC	2.96	11.04	8.63	21.24	6.24	13.85	12.28	25.44	13.72	31.11	28.10	63.71	6.60	13.28	11.94	24.02
Omaha, NB	2.59	10.29	7.96	19.89	6.10	13.57	12.03	24.94	12.08	27.38	24.73	56.07	7.87	15.84	14.24	28.66

*Corrected for combustion effeciency as follows:

Gas = .75
Oil = .60
Electric Resistance = 1.00
Heat Pump = variable COP by location

**The Annualized cost in year t´ is defined as $A_{t'} = CR \times \sum_{t=1}^{T} \left(\frac{1}{1+i}\right)^t C_{(t+t')}$

where:

$C_{t'}$ = current delivered cost ($\$/10^6$Btu) in year t´
i = nominal discount rate = r + AIR
T = system life in years
CR = capital recovery factor = $\frac{i}{1 - \left(\frac{1}{1+i}\right)^T}$
t´ = 1,13 (1978-1990)

$A_{t'}$ = annualized delivered cost ($\$/10^6$ Btu) in year t´
r = real discount rate
AIR = annual inflation rate

Values used in the derivation of these figures are as follows:

r = .035
AIR = .06
i = .095

T = 30
CR = .102 (This assumes mortgage & nominal discount rates are identical.)

TABLE VII

SELECTED FINANCIAL INDICATORS FOR COMPARATIVE EVALUATION OF SOLAR SYSTEMS
(Six Alternative Cases For Six Representative Sites)

	Natural Gas					Electric Resistance				
Alternative Case Number*	Year	Optimal Solar Fraction	Mortgage Payback	Years to Positive Savings	Net Present Value	Year	Optimal Solar Fraction	Mortgage Payback**	Years to Positive Savings**	Net Present Value***
CASE 1										
Albuquerque, NM						1978	.70	16	1	4369
Madison, WI						1978	.50	14	0	6341
Boston, MA			NO FEASIBILITY			1978	.55	14	0	6799
Seattle, WA						--	--	--	--	--
Charleston, SC						1982	.60	22	9	653
Omaha, NB						1978	.55	15	1	5393
CASE 2										
Albuquerque, NM						1978	.40	13	0	3033
Madison, WI						1978	.25	16	2	2658
Boston, MA			NO FEASIBILITY			1978	.30	16	2	3034
Seattle, WA						--	--	--	--	--
Charleston, SC						1978	.45	14	0	1649
Omaha, NB						1978	.25	15	1	2403
CASE 3										
Albuquerque, NM						1978	.55	13	0	3991
Madison, WI						1978	.45	16	1	4833
Boston, MA			NO FEASIBILITY			1978	.45	15	0	4853
Seattle, WA						--	--	--	--	--
Charleston, SC						1978	.60	15	1	2000
Omaha, NB						1976	.50	16	2	4392
CASE 4										
Albuquerque, NM	1978	.10	20	8	77	1978	.65	11	0	5333
Madison, WI	1978	.10	18	8	288	1978	.45	12	0	6354
Boston, MA	1978	.10	19	8	206	1978	.50	12	0	6579
Seattle, WA	1978	.10	20	8	111	1978	.10	21	8	98
Charleston, SC	1981	.10	21	6	44	1978	.60	11	0	2652
Omaha, NB	1984	.10	21	7	103	1978	.58	12	0	5626
CASE 5										
Albuquerque, NM	1978	.15	23	8	122	1978	.70	14	0	8317
Madison, WI	1978	.15	22	8	469	1978	.45	13	0	9850
Boston, MA	1978	.10	22	8	245	1978	.50	14	0	10212
Seattle, WA	1978	.10	23	6	123	1976	.10	24	11	84
Charleston, SC	1979	.10	24	9	23	1978	.70	15	0	4121
Omaha, NB	1982	.10	24	10	89	1978	.50	14	0	9023
CASE 6										
Albuquerque, NM	1978	.35	20	8	726	1978	.75	12	0	10079
Madison, WI	1978	.30	20	8	1407	1978	.45	11	0	11587
Boston, MA	1978	.35	21	8	1136	1978	.50	11	0	12025
Seattle, WA	1978	.20	20	8	592	1978	.20	21	7	513
Charleston, SC	1976	.35	21	8	277	1978	.75	12	0	5160
Omaha, NB	1978	.20	22	8	450	1978	.50	11	0	10659

-- Indicates no feasibility.

* SYSTEM	NIGHT INSULATION	TAX CREDITS	LOW INTEREST LOAN
CASE 1: Air Collector/Rock Storage	No	Yes	No
CASE 2: Trombe wall	No	No	No
CASE 3: Trombe wall	Yes	No	No
CASE 4: Trombe wall	Yes	Yes	No
CASE 5: Trombe wall	Yes	No	Yes
CASE 6: Trombe wall	Yes	Yes	Yes

See text for appropriate values.

**Expressed in years.

***Expressed in "year of feasibility" dollars

F. R.

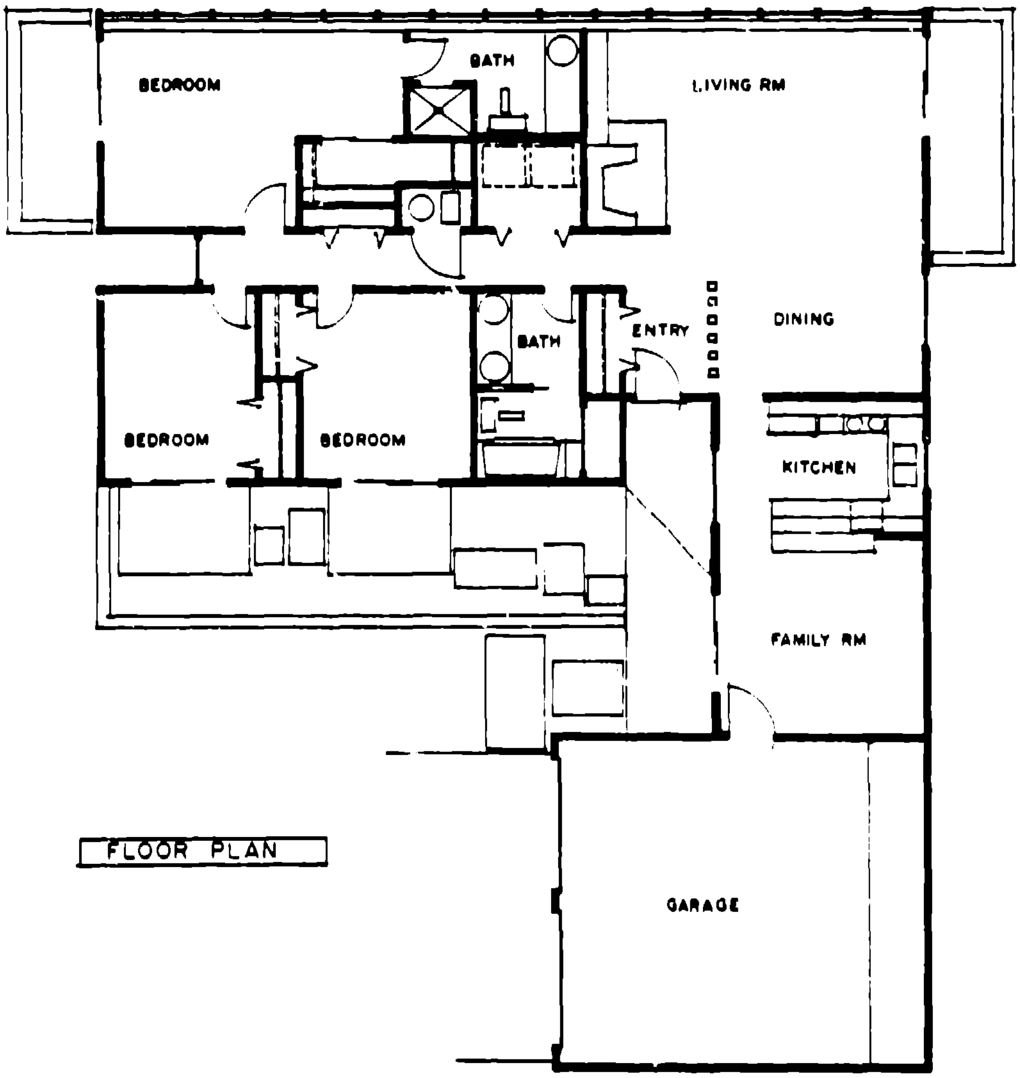


FIG. 1. TRACT HOME CONCEPT USED IN SOLAR DESIGNS.

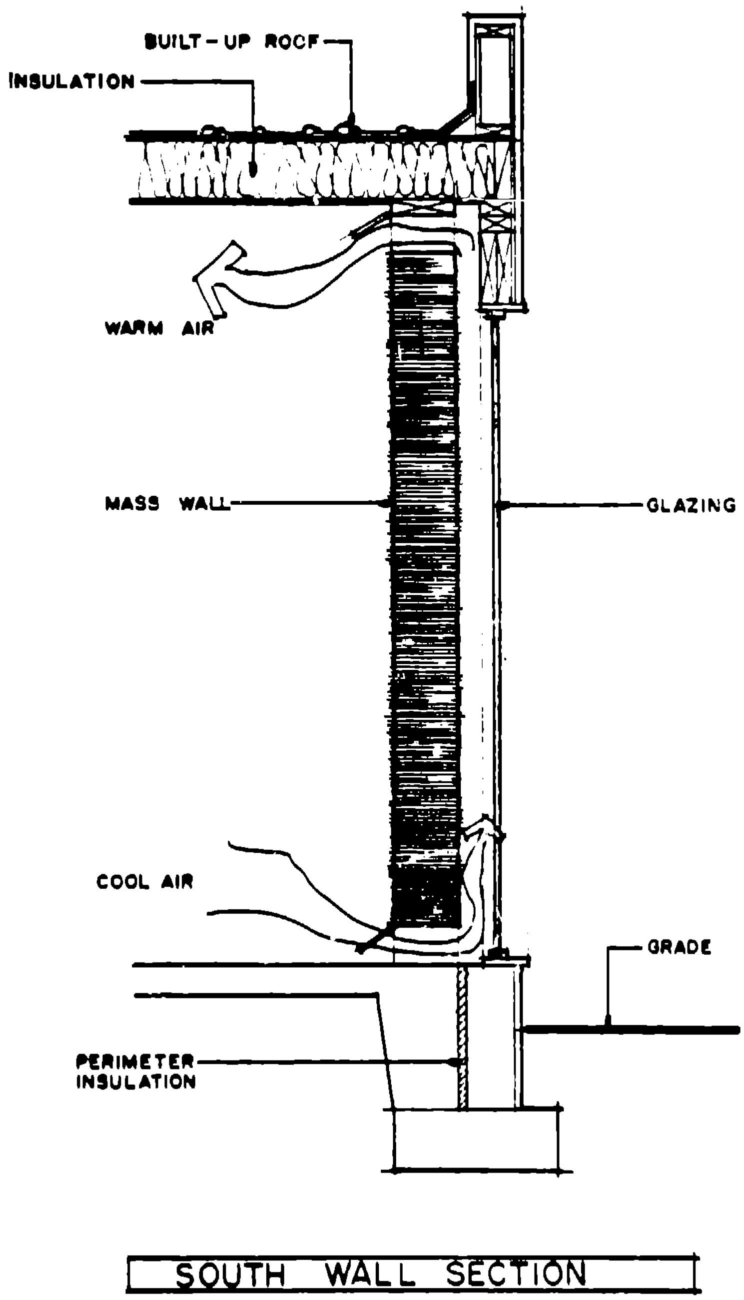
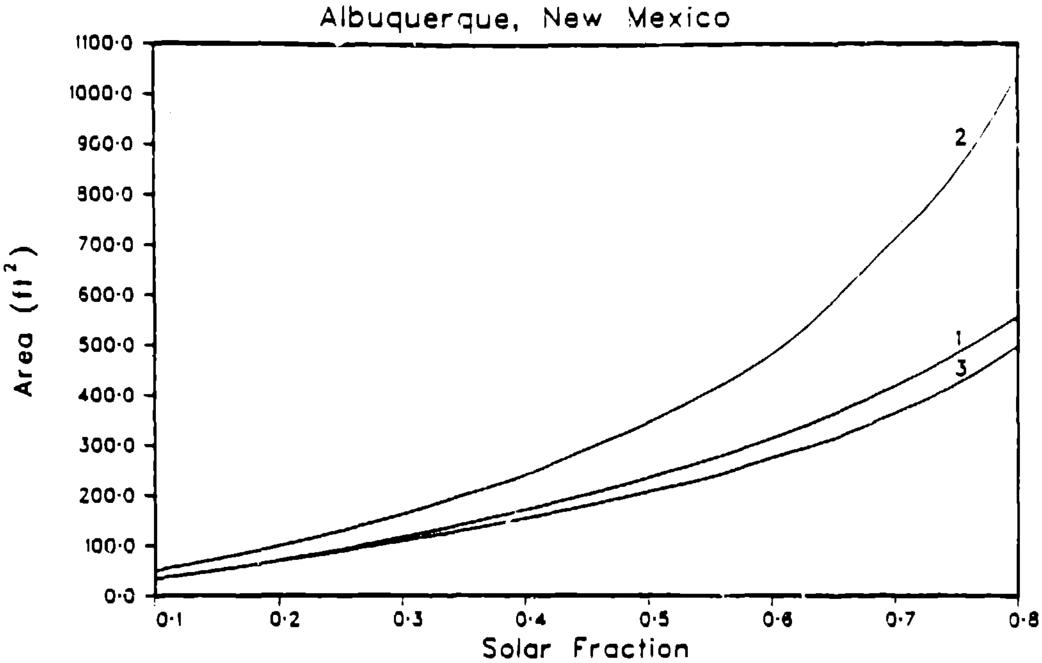
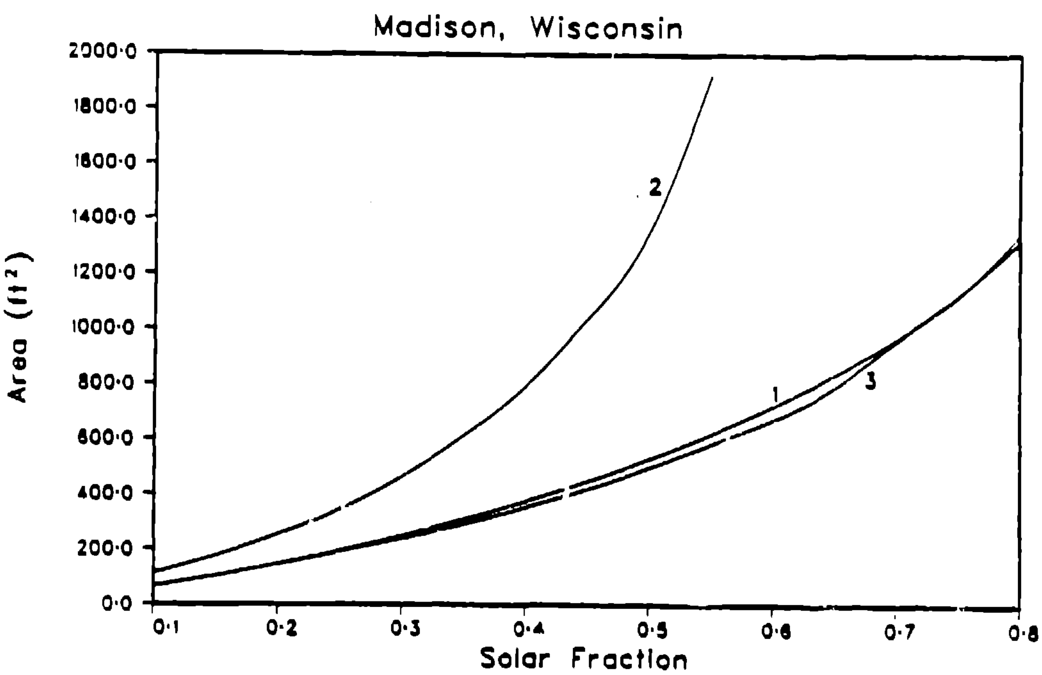


FIG. 2. DETAILED ARCHITECTURAL CHARACTERISTICS OF TROMBE WALL.



1 = Air Collector/Rock Storage
2 = Trombe Wall w/o Night Insulation
3 = Trombe Wall with Night Insulation

FIG. 3A. GLAZING (COLLECTOR) AREA REQUIRED (FT^2) SOLAR FRACTION



1 = Air Collector/Rock Storage
2 = Trombe Wall w/o Night Insulation
3 = Trombe Wall with Night Insulation

FIG. 3B. GLAZING (COLLECTOR) AREA REQUIRED (FT^2) SOLAR FRACTION

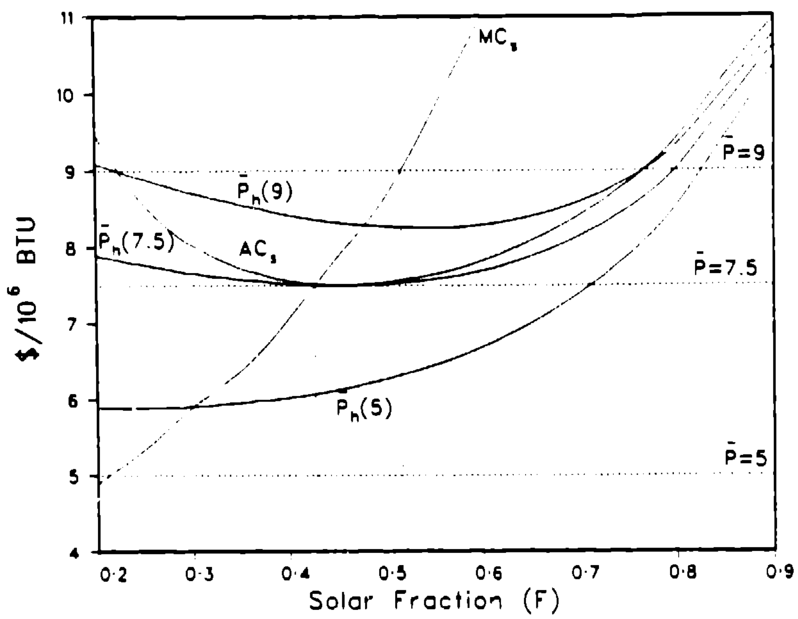


FIG. 4. ACTIVE SYSTEM WITH FIXED COSTS - MARGINAL, AVERAGE, AND DELIVERED HEAT COST

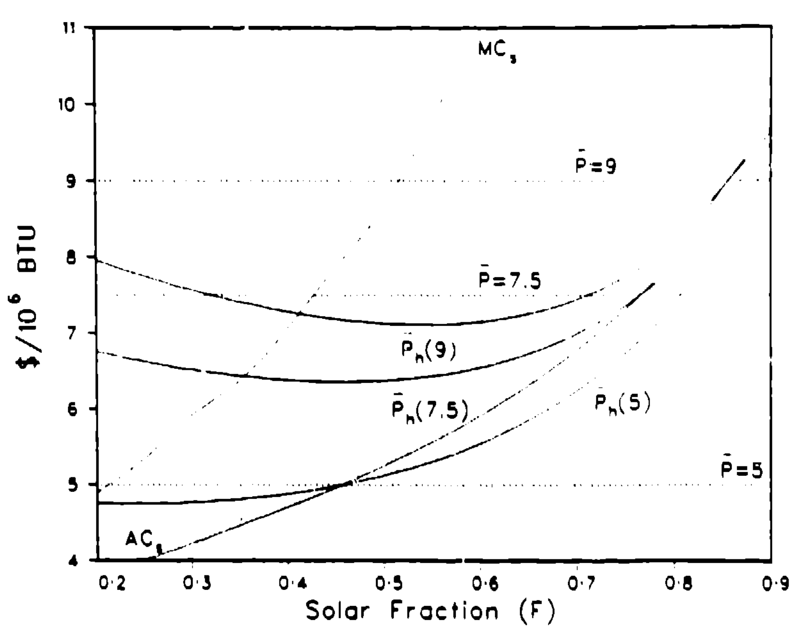


FIG. 5. PASSIVE SYSTEM WITHOUT FIXED COSTS - MARGINAL, AVERAGE, AND DELIVERED HEAT COST

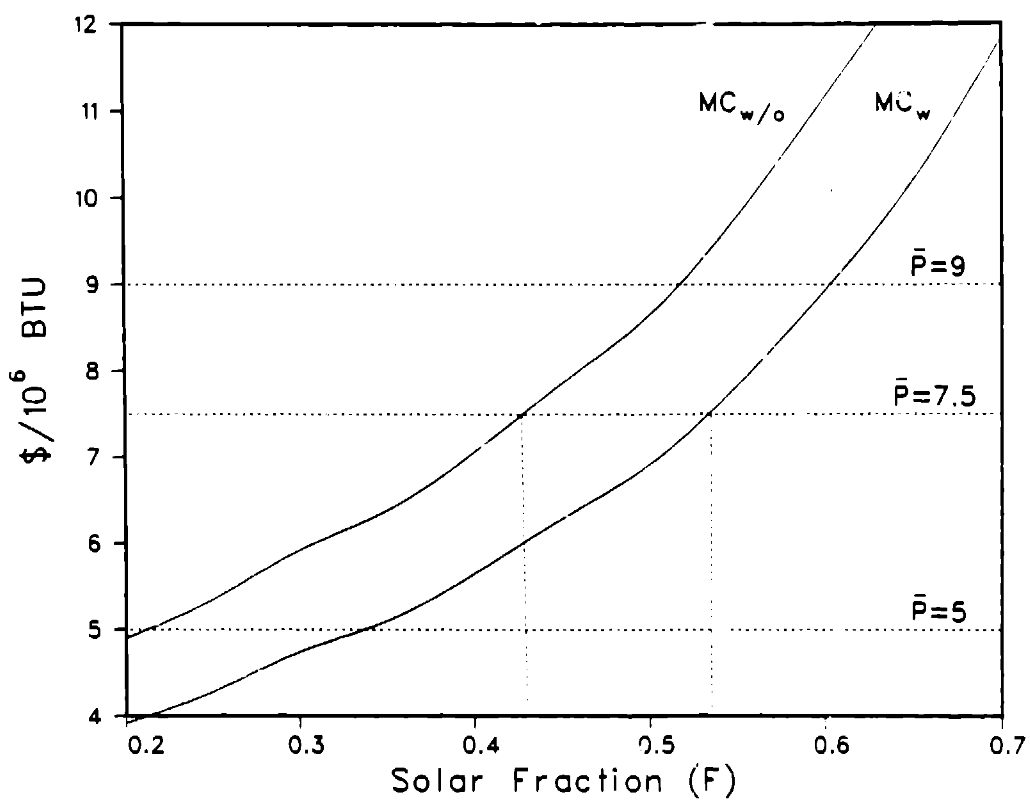
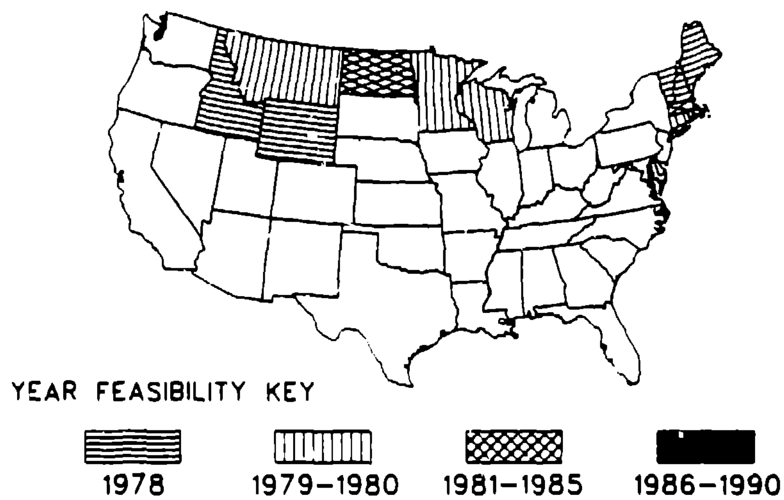


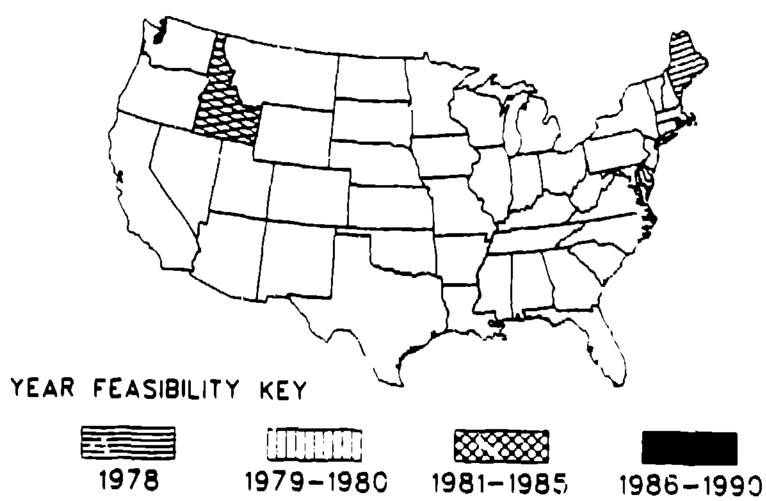
FIG. 6. THE EFFECT OF SOLAR INCENTIVES ON MARGINAL COST AND OPTIMAL SIZING.

SOLAR FEASIBILITY FOR AIR COLLECTOR/ROCK STORAGE ACTIVE SYSTEM
ALTERNATIVE FUEL - NATURAL GAS
NEA TAX CREDIT AND LOW INTEREST LOAN INCENTIVES
(30-YEAR LIFE CYCLE COST BASIS)



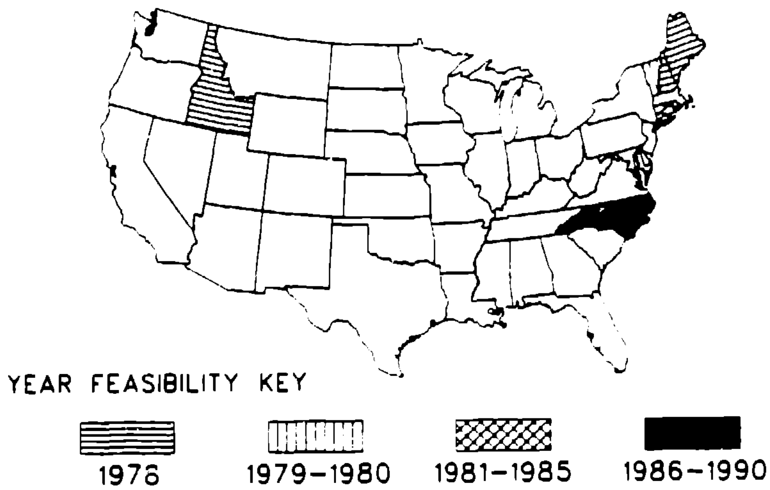
MAP 1

SOLAR FEASIBILITY FOR TROMBE WALL W/O NIGHT INSULATION
ALTERNATIVE FUEL - NATURAL GAS
NO INCENTIVES
(30-YEAR LIFE CYCLE COST BASIS)



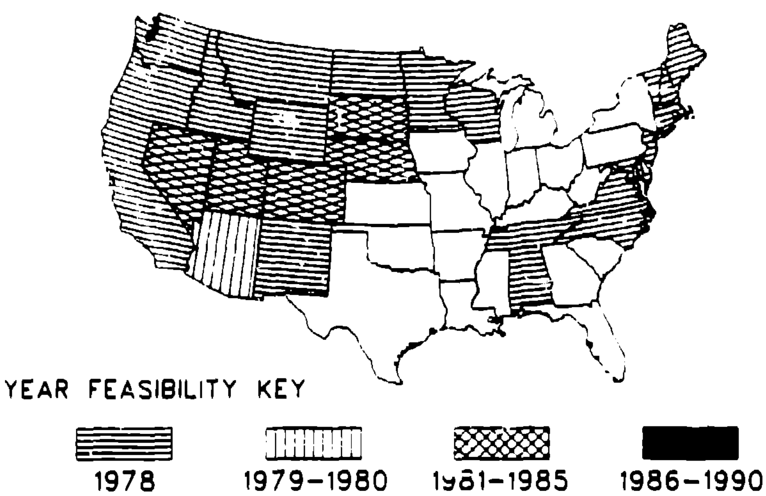
MAP 2

SOLAR FEASIBILITY FOR TROMBE WALL WITH NIGHT INSULATION
ALTERNATIVE FUEL – NATURAL GAS
NO INCENTIVES
(30–YEAR LIFE CYCLE COST BASIS)



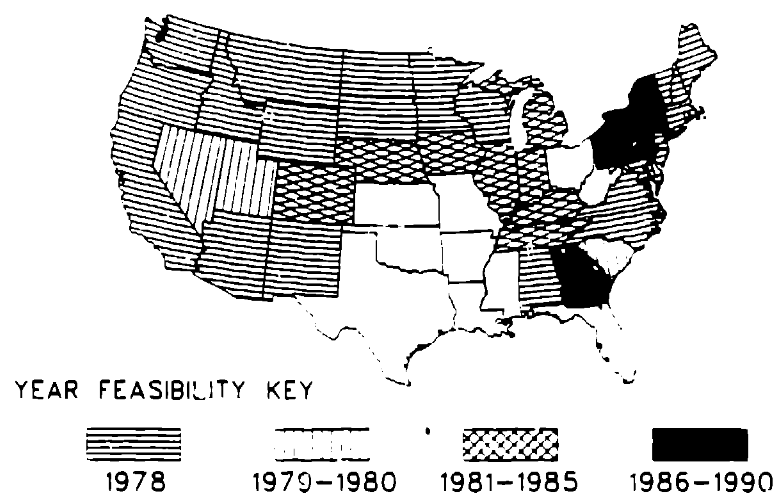
MAP 3

SOLAR FEASIBILITY FOR TROMBE WALL WITH NIGHT INSULATION
ALTERNATIVE FUEL – NATURAL GAS
NEA TAX CREDIT INCENTIVE
(30–YEAR LIFE CYCLE COST BASIS)



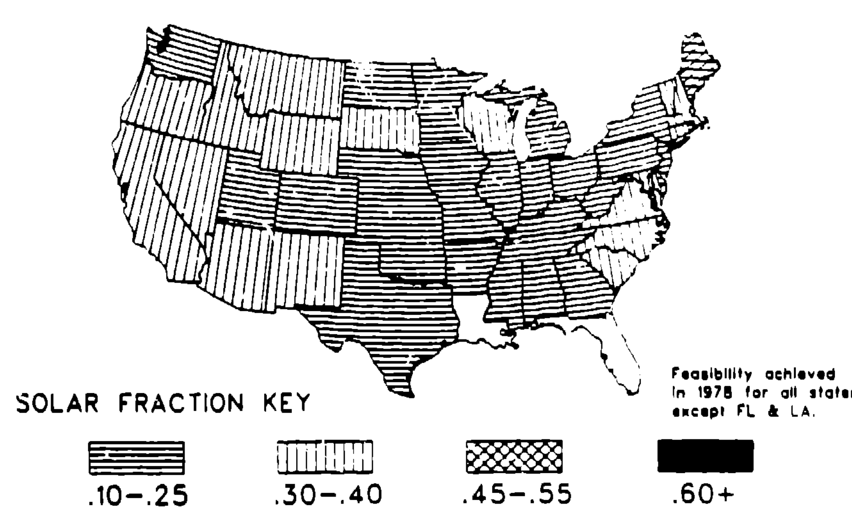
MAP 4

SOLAR FEASIBILITY FOR TROMBE WALL WITH NIGHT INSULATION
ALTERNATIVE FUEL – NATURAL GAS
LOW INTEREST LOAN INCENTIVE
(30-YEAR LIFE CYCLE COST BASIS)



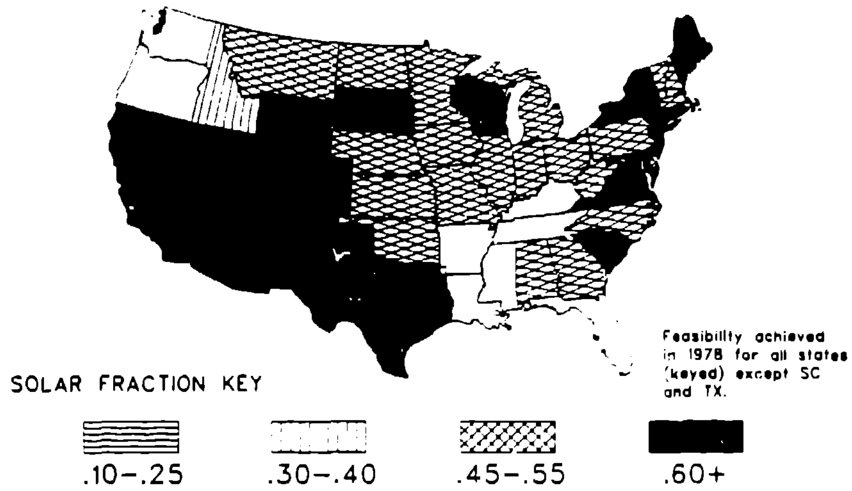
MAP 5

SOLAR FEASIBILITY FOR TROMBE WALL WITH NIGHT INSULATION
ALTERNATIVE FUEL – NATURAL GAS
NEA TAX CREDIT AND LOW INTEREST LOAN INCENTIVES
(30-YEAR LIFE CYCLE COST BASIS)



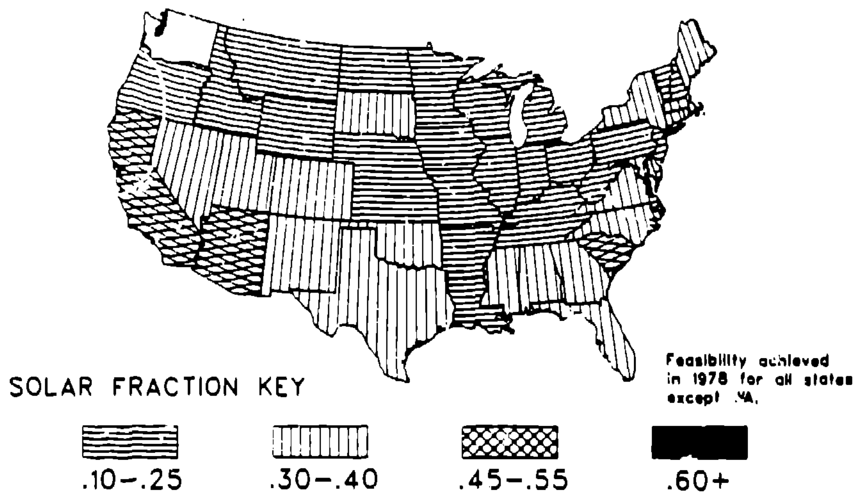
MAP 6

SOLAR FEASIBILITY FOR AIR COLLECTOR/ROCK STORAGE ACTIVE SYSTEM
ALTERNATIVE FUEL - ELECTRICITY (RESISTANCE)
NEA TAX CREDIT INCENTIVE
(30-YEAR LIFE CYCLE COST BASIS)



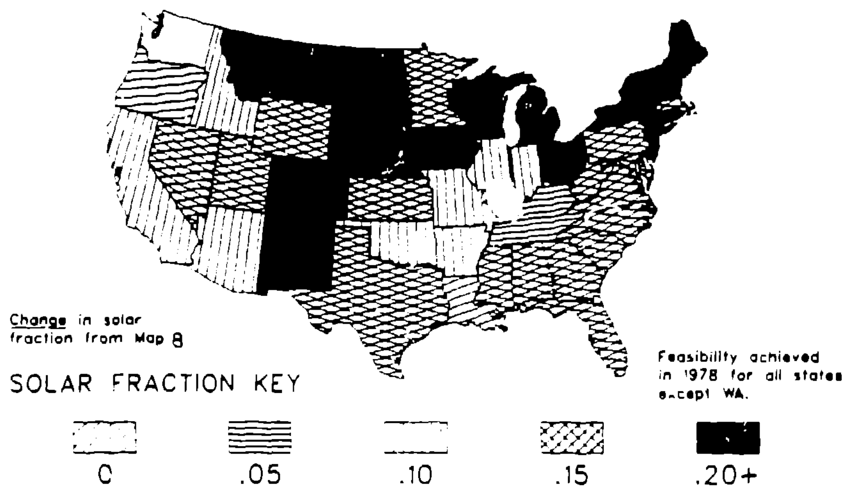
MAP 7

SOLAR FEASIBILITY FOR TROMBE WALL W/O NIGHT INSULATION
ALTERNATIVE FUEL - ELECTRICITY (RESISTANCE)
NO INCENTIVES
(30-YEAR LIFE CYCLE COST BASIS)



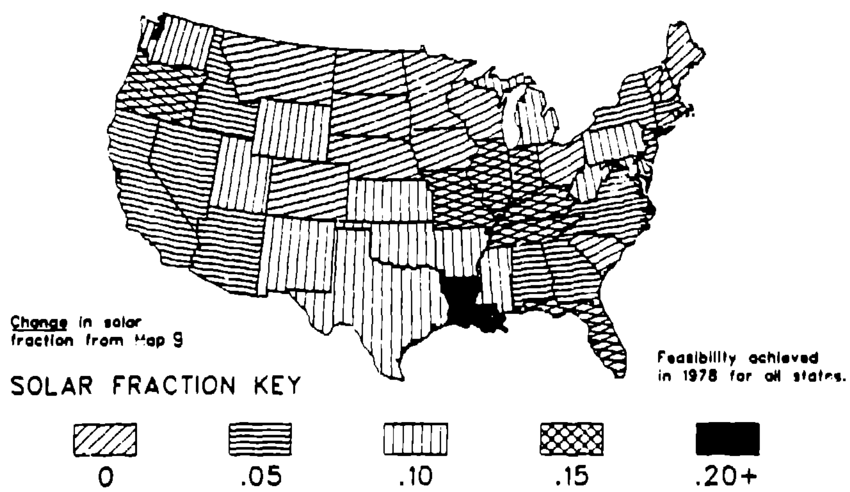
MAP 8

SOLAR FEASIBILITY FOR TROMBE WALL WITH NIGHT INSULATION
ALTERNATIVE FUEL – ELECTRICITY (RESISTANCE)
NO INCENTIVES
(30-YEAR LIFE CYCLE COST BASIS)



MAP 9

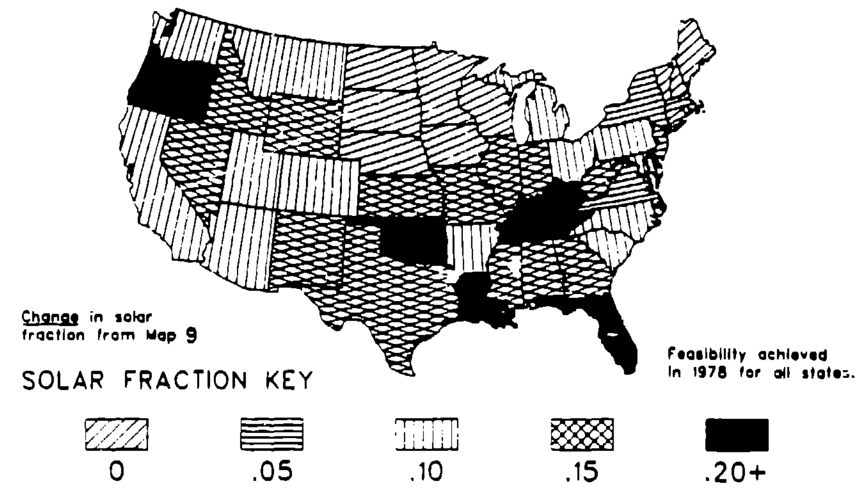
SOLAR FEASIBILITY FOR TROMBE WALL WITH NIGHT INSULATION
ALTERNATIVE FUEL – ELECTRICITY (RESISTANCE)
NEA TAX CREDIT INCENTIVE
(30-YEAR LIFE CYCLE COST BASIS)



MAP 10

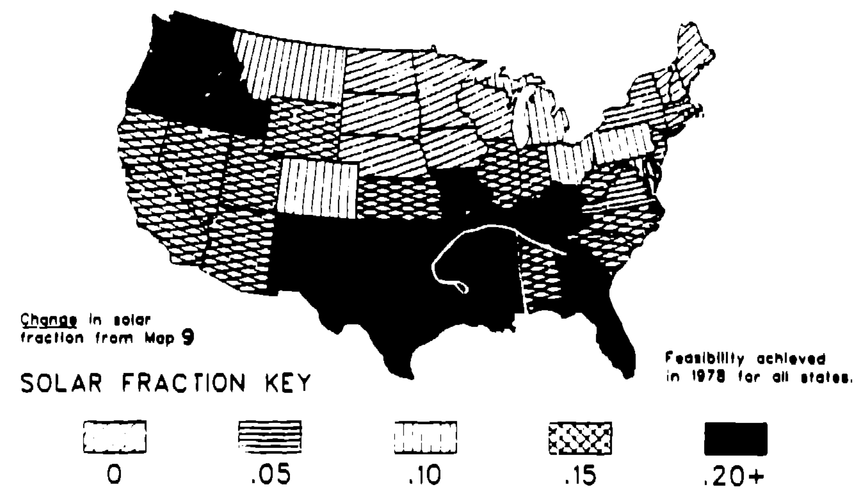
F. R.

SOLAR FEASIBILITY FOR TROMBE WALL WITH NIGHT INSULATION
ALTERNATIVE FUEL - ELECTRICITY (RESISTANCE)
LOW INTEREST LOAN INCENTIVE
(30-YEAR LIFE CYCLE COST BASIS)



MAP 11

SOLAR FEASIBILITY FOR TROMBE WALL WITH NIGHT INSULATION
ALTERNATIVE FUEL - ELECTRICITY (RESISTANCE)
NEA TAX CREDIT AND LOW INTEREST LOAN INCENTIVES
(30-YEAR LIFE CYCLE COST BASIS)



MAP 12